

Calculators are not allowed

Answer the following questions.

1. Let $f(x) = \sin^7(5x^3 - 2x + 1)$. Find $f'(x)$ (5 pts)

2. Use Rolle's theorem to show that the function $f(x) = x \cos x$ has a critical number in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. (3 pts)

3. Use differentials to approximate the value of $\sqrt[3]{28}$. (5 pts)

4. Find the two real numbers whose sum is 100 and their product is maximum. (5 pts)

5. Find $f'(x)$, if

$$f(x) = \int_{5x}^{x^2} \sin t^2 dt + \int_0^3 \sqrt{t^2 + 16} dt$$
 (5 pts)

6. Evaluate the following integrals:

(a) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^{10}}$

(b) $\int_0^1 (x^2 + 2x + 1)^5 dx$. (7 pts)

7. Let $f(x) = \frac{x+1}{x+3}$.

(a) Find the intervals on which f is increasing or decreasing and find the local extrema of f (if any).

(b) Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any)

(c) Find the horizontal and vertical asymptotes for the graph of f (if any).

(d) Sketch the graph of f . (10 pts)

(Good Luck)